

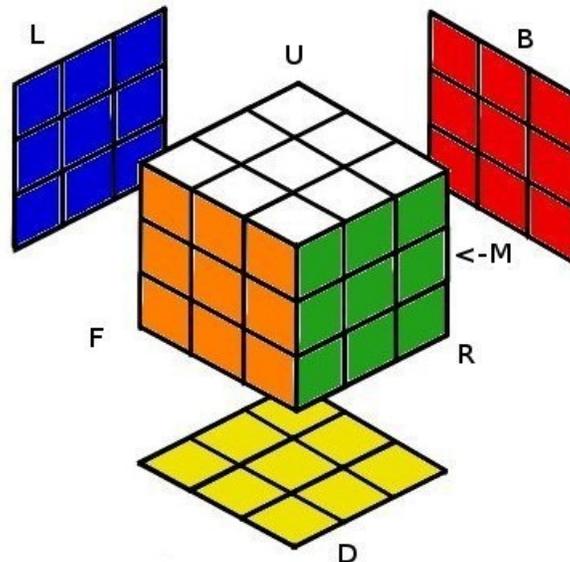
# Rubik's 3x3x3 Cube

Donna A. Dietz  
American University  
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This paper is the second of a pair of papers, the first of which concerns my 2x2x2 cube solution<sup>1</sup>. The reader should understand that paper prior to reading this. Once the ideas in these papers are understood, the reader could use the underlying strategies to generate solutions to the Pyraminx or Skewb or even a larger Rubik's cube “from scratch”. It may be easier to watch my YouTube videos rather than read this in print. <http://www.youtube.com/profdietzAU>

The first step of the 3x3x3 cube will be to solve the corners of the cube using the techniques in the previous paper. The general flow of the solution will be to first solve the corners, then to place the edges of the Up and Down faces, in pairs belonging to the same face, then to place the edges of the middle slice, and finally to flip pairs of edges until the cube is fully solved.

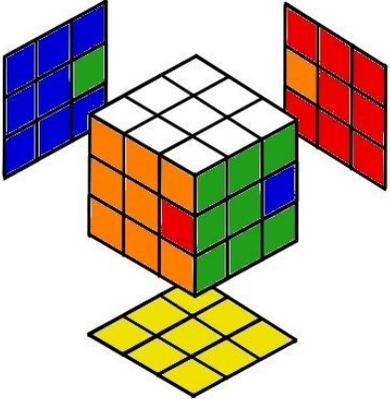
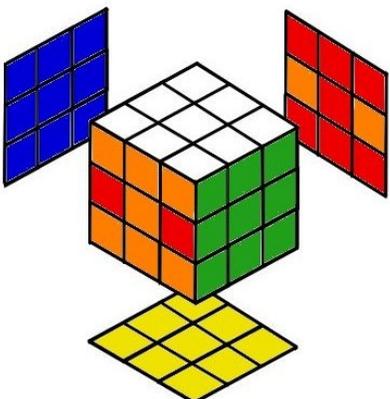
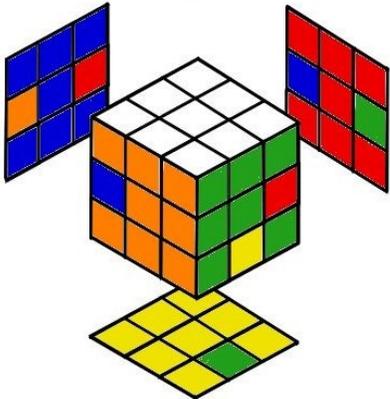
The notation is the same as the previous paper except for the addition of two “moves”, for rotating the middle slice clockwise or counterclockwise. The middle slice to be rotated is the one which is between the Up and the Down faces, and it rotates clockwise (M) or counterclockwise (m) as the cube is viewed from the Up face.

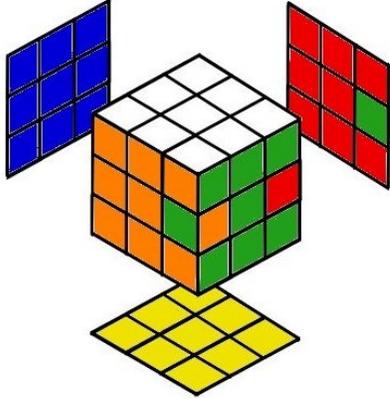


As before, we ask, “When I do a simple repeated motion starting with a solved cube, how long does it take to get back to the solved state, and what fun and interesting things happen along the way?” As we do this, we keep track of the results and write them down for future use. I will leave it as an exercise for the reader, and I will just record the routines we will be making use of.

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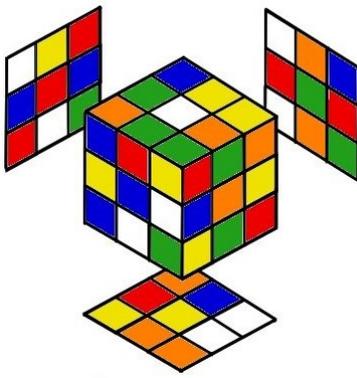
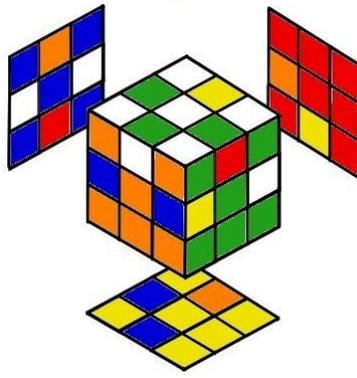
<sup>1</sup> Both this and the previous paper are under construction. The first paper has been replaced for the moment with three handouts from class, for stages one, two, and three of the 2-cube solution.

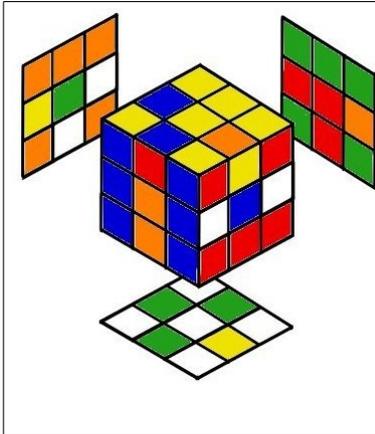
Routine	Repetitions to re-solve	Fun stopping points	What it looks like
RRMRRm	3	1	
(RRMM) <sup>2</sup>	2	1	
(RM) <sup>4</sup>	2	1	

$(RM)^4(MR)^4$	2	1	
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The routines above will be used in the final stages of the solution, but before we can make use of them, we will want to place all the edges in the Up and Down faces, although they may be flipped. This part should become intuitive fairly quickly, and as with the 2x2x2, I will rely on an example as the basis for my discussion of the strategies for doing this.

This process of placing Up/Down pairs of edges, I call “setting up the totem poles”, because, just as totem poles are constructed on the horizontal, then transported horizontally, and finally erected, we will also set up and construct these pairs horizontally and then finally set them erect in their final resting spots.

	First, I scrambled the cube!
	Then, I used the techniques from the 2x2x2 cube to solve the 8 corners of this cube. I also made sure to set the axis in its correct location relative to the corners. It suffices at this stage to just set the Up and Down centers, but for aesthetics, I set all of them.

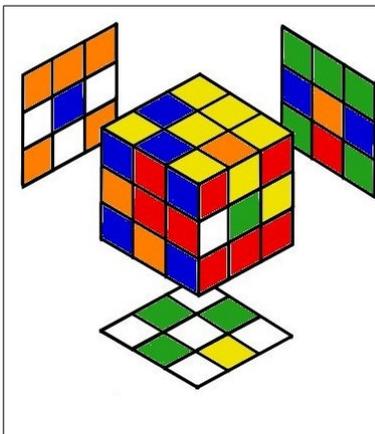


A this stage, we begin to build and erect “totem poles”. I inspected the middle layer and noticed that there was already a totem pole! In the previous diagram, on the Front, notice the Right and Left edges are a pair of blue edges which should be on Up/Down. They are already adjacent to one another. So, I rotated that pair (totem pole) using M or m turns, until it was on its own color-face (Blue), just so I could see which way it was oriented. (I also inadvertently turned the cube upside-down, which didn't matter one bit.) I noticed that the Blue-White edge was on the Right, while the Blue-Yellow was on the Left. To remind myself, I said, “White on the Right”.

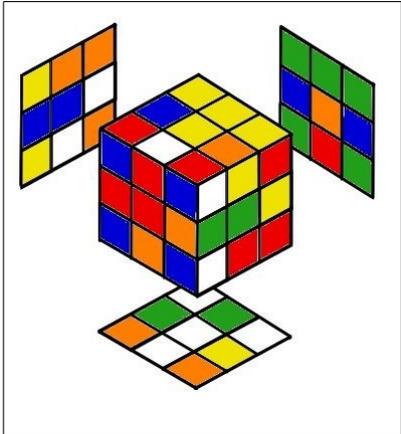
This "totem pole" idea is not mine, but I may be the first to give it this name. I have had trouble explaining this concept to my students, but once they get it, it makes sense, and they keep calling it that, so I have no other name for this concept.

The point is that we are placing a pair of edges which live on opposite sides of a given face. We may do this repeatedly as long as we leave one middle layer (in this case, the middle layer parallel to the floor) free to hold any leftovers or "garbage" we don't care about for the moment. I call this the garbage layer. We use this garbage layer to temporarily hold the pair of edges we wish to place, but when we place them in the garbage layer, we only require that they be neighbors (not catty-corner). This is analogous to carving a totem pole, because likewise, a real totem pole must be constructed on the horizontal prior to being erected into its vertical position. We imagine the middle of the face as being part of the totem pole, but it really only comes back after the edges are set. After erecting a totem pole, I usually reset the axes just as an error-correction step.

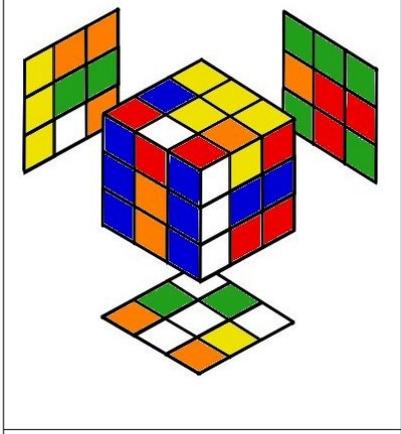
Once the 8 corners are set, we will only slightly move them in this stage, in order to set up the 4 top and the 4 bottom edges. In each case, we may move a Front face (any non-Up, non-Down face) 90 degrees, do what we need to do by only moving the middle layer, and then undo the 90 degree turn.



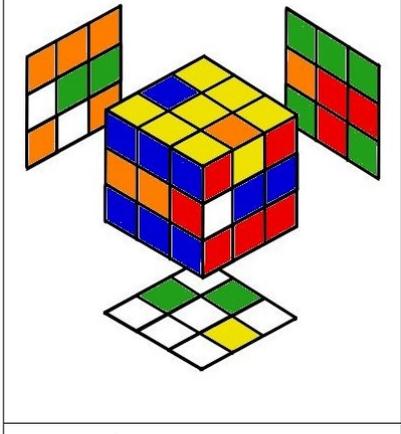
I then rotated the middle layer until the totem pole was safely on the Back, relative to the color face where the totem pole is to be erected. (Two quarter turns in either direction.)



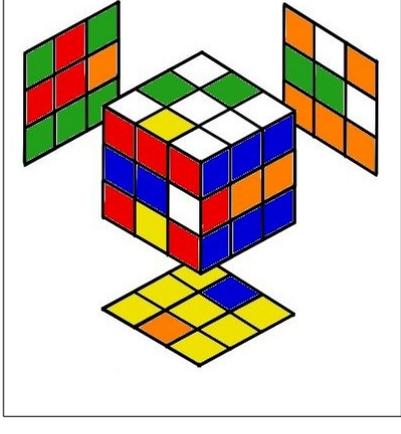
If “White is on the Right”, I rotate the Front face clockwise. (Otherwise, I would rotate the Front face counterclockwise.) There is now a “frame” ready to receive the totem pole, which should be transported horizontally from the Back by rotating the middle layer two turns.



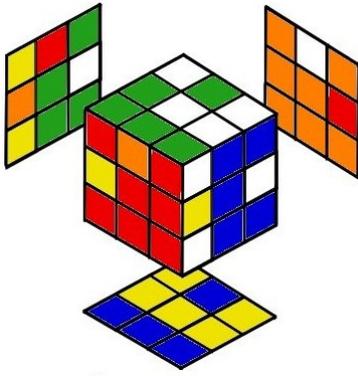
The totem pole is now in its “frame” and is ready to be erected. So, I turned Front back into its usual position, complete with the totem pole!



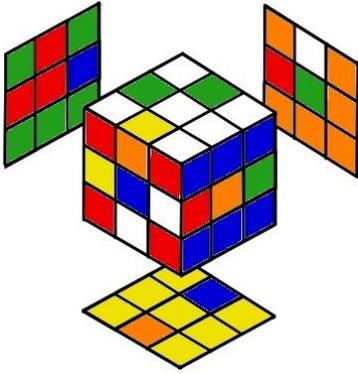
Like this.



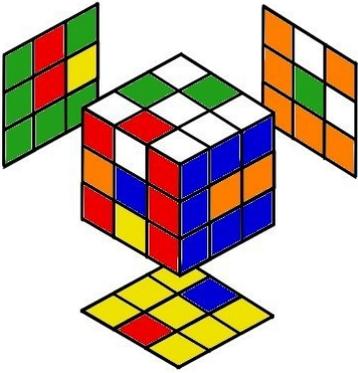
I then realized flipped the cube back into its usual orientation with white on the Up layer. The next totem pole I decided to set up was the red one. The Red-White edge was already in the middle, but the Red-Yellow needed to be dislodged from the Up layer. So, I moved the middle layer one turn counterclockwise to protect the Red-White edge, then turned the Front face clockwise so that the Red-Yellow and Red-White edges would be adjacent.



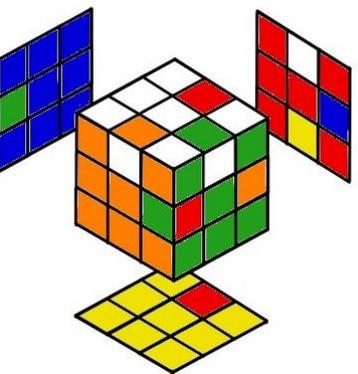
Like this. I moved this new Red totem safely to the back of the cube while I righted the front face, and then brought the Red totem back to the front of the cube with two quarter turns of the middle layer.



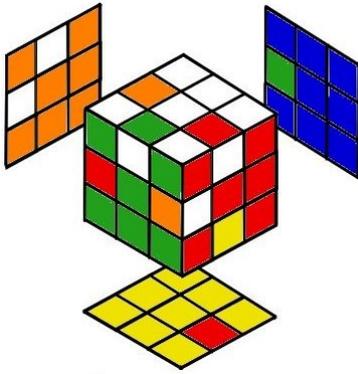
Like this! I saw I had white on the right again, so I repeated the same thing I did before, and set up the Red totem.



Like this!

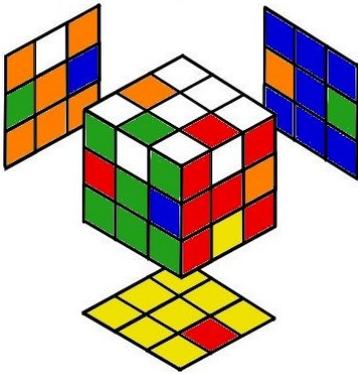


I used the same techniques to set up the remaining two totems. Here, I see that the next step is to place the edges of the middle layer. There are 3 misplaced edges in the middle layer, but we are brushing that under the rug in this solution, relying instead on having 4 misplaced edges at this stage! So, we have a little “workaround” routine to force the good edge out of its location.

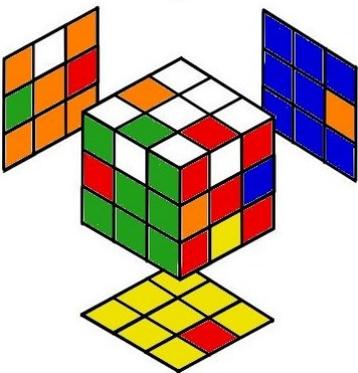


Rotate the cube so that the RB edge is the correct edge, and use the following routine once (if that is all that is needed) or twice (if the correct “moves” to LB). If you need to use the routine twice, don't change the orientation of the cube between iterations.

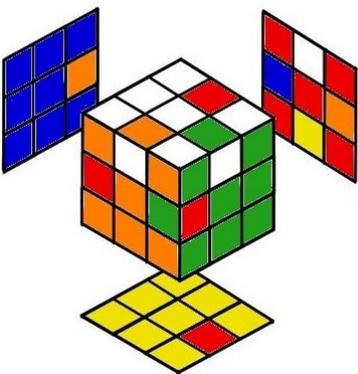
$R^2 M R^2 m$



After using the little routine once, the “correctness” has moved to LB, so we repeat the routine.

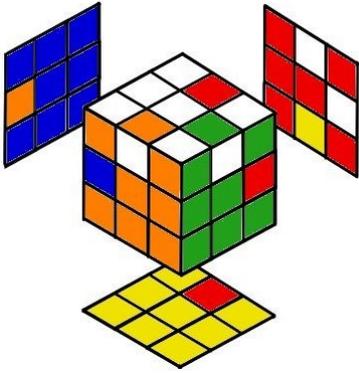


We have now successfully messed up all 4 edges of the middle slice! There are two cases. Either they are in an “X” shape, relative to their actual positions, or they are swapped with neighbors, making two parallel lines. You just look at any corner and ask “Is either color correct for this incorrect edge?” If it is, you have parallel swaps. Otherwise, you have an “X” shape, and the first round of this routine can be done with any orientation, so long as your Up and Down faces remain Up and/or Down. The second round will have to be a set of parallel swaps.

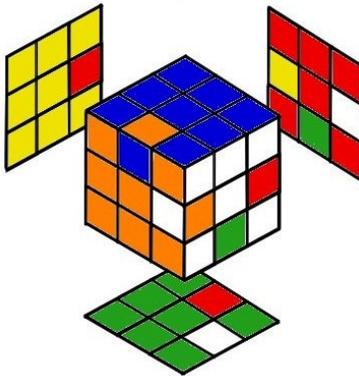


If you already have need of two parallel swaps, as I did, reorient the cube so as to place a pair to be swapped on the Right (RF, RB) and a pair to be swapped on the Left (LF, FB).

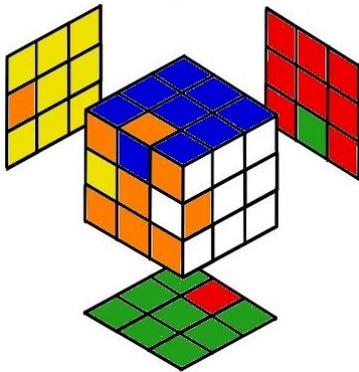
$R^2 M^2 R^2 m^2$



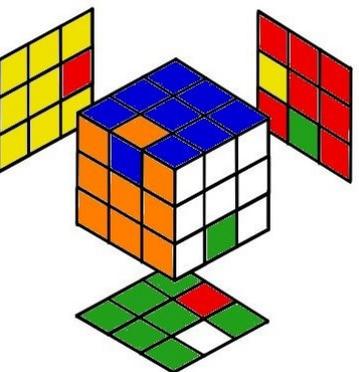
We are nearly done, as all edges are where they belong and need only to be flipped! In the Rubik's cube, flipped edges only happen in pairs. So, we will use  $(RM)^4(MR)^4$  as many times as we need, starting with pairs needing to be flipped at RF/RB.



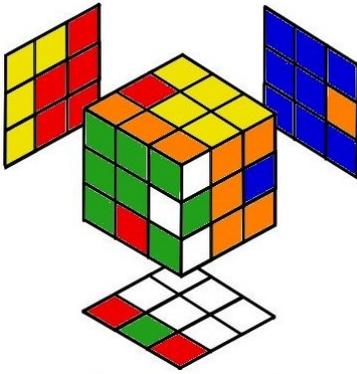
For example, this is the first orientation I picked.



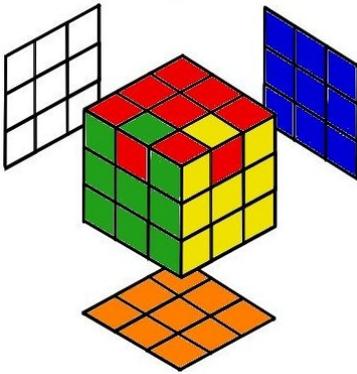
This is what the cube looked like, halfway through the routine! You can see that most of the action is in the middle layer, but one edge on the Down face has flipped, for a total of 4 flipped edges.



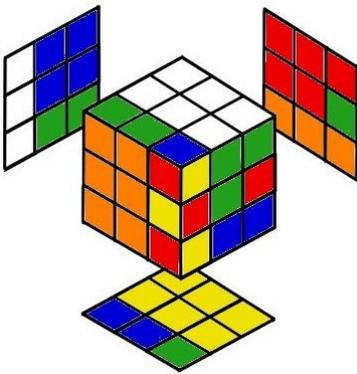
Finishing the routine simply flips the two edges as promised. But wait! There are no more pairs to flip which are naturally across from each other! So, we use a setup move! In this diagram, white is on the Right and green is Down. I rotated Down a quarter turn counterclockwise, so that the white-green edge was across from the blue-orange edge.



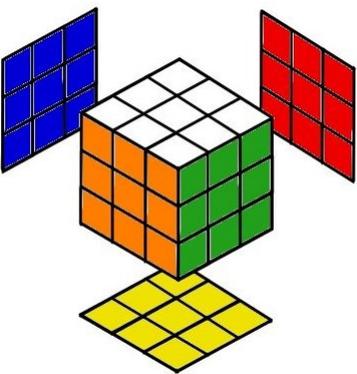
I then reoriented the cube so that my right hand held the side of the cube with both edges which I wanted to flip.



The routine worked as always, and the setup move was reversed, bringing us to this common state of needing yet more setup moves to get the two remaining edges to opposing sides of the same face. In this case,  $R^2d$  did the trick.



As awful as this looks (now, and especially during the routine), we are just about done with this cube! The routine works as it has been doing, and the setup moves were reversed, bringing us to....



The solved cube! Finally!

All the routines and logical steps needed for solving the 3x3x3 cube have been discussed in this demonstration, although you can feel free to rearrange the steps and make up your own routines. For example, you may actually prefer the RRMRRm routine which moves three edges of the middle slice, and you may not like the parallel swap move. Or, perhaps you don't mind memorizing a few extra routines. You can do whatever you like! That's the whole beauty of this solution! You don't have to use any routines you don't like! You just find routines you feel natural with and go with it. In the final analysis, there are so many different ways to solve the cube, you have the freedom to pick and choose. Once you realize you can solve the cube without staring at any paper or without memorizing 25 routines or cases, you may just find yourself temporarily addicted or re-addicted to this toy!

HAPPY CUBING!